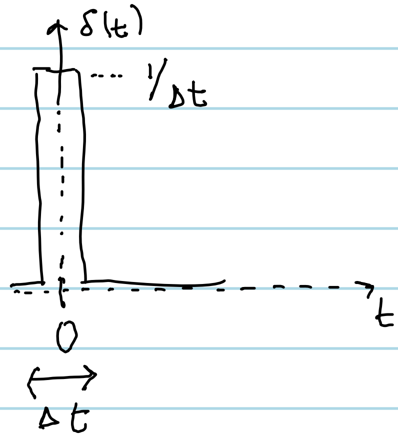
Delta function definition boardwork

Delta function



$$\delta(t) = \lim_{\Delta t \to 0} \begin{cases} 1/\Delta t & \text{if } -\Delta t/2 < t < \Delta t/2 \\ 0 & \text{otherwise} \end{cases}$$

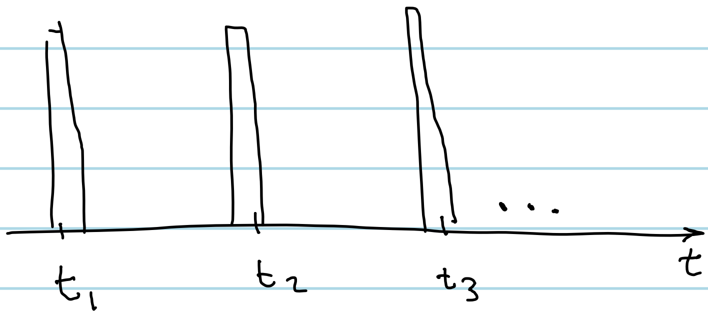
$$\delta(t - t_i) = \lim_{\Delta t \to 0} \begin{cases} 1/\Delta t & \text{if } t_i - \Delta t/2 < t < t_i + \Delta t/2 \\ 0 & \text{otherwise.} \end{cases}$$

"Spike at time t_i"

$$\int dt \; \delta(t - t_i) = \Delta t \cdot \frac{1}{\Delta t} = 1 \qquad \text{(rectangle area)}$$

Any interval containing t_i

Spike train: $\rho(t) = \sum_{i=1}^{n} \delta(t - t_i)$



$$\#\text{ spikes in } \Delta T = \int_0^{\Delta T} \rho(t)\,dt$$

So.... An estimate of rate $= \dfrac{\#\text{ spikes in } \Delta T}{\Delta T}$

$$r = \frac{1}{\Delta T}\int_0^{\Delta T} \rho(t)\,dt$$

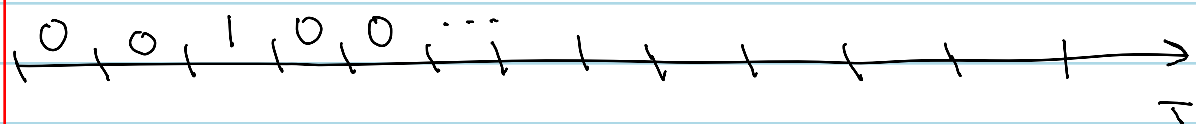
or... Time-Dependent rate $\quad r(t) = \dfrac{1}{\Delta T}\displaystyle\int_t^{t+\Delta t} \rho(t')\,dt'$

- Modeling a spike train as a Poisson Process:

- Divide time axis into BINS of width Δt [N.B. different use of Δt than for δ fn definition]

- Rate r. So... spike in each bin with proba. $r \cdot \Delta t = p$

- Put 1 if spike, 0 otherwise



- Generating (samples of) BINARY (0/1) random variables in MATLAB.

BASICS of PROBABILITY... following Cox's Gabbiani Ch. 11

def: RANDOM VARIABLE: object X defined by:

1) set of possible values (or states) $\{a_1, \ldots, a_N\}$

2) probability distribution defined over sample space

def DISCRETE RANDOM VARIABLE: random variable with DISCRETE values

def: Realization: random assignment of X to one of its values, with specified probabilities

INTERPRETATION: For discrete random var., proba. $p_j(a_j)$

- COIN TOSSING AND BINARY-VALUED RANDOM VARIABLES:

values $\{a_1, a_2\} = \{1, 0\}$

PROBABILITIES:

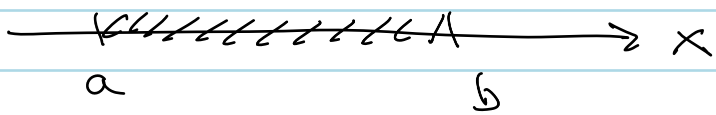
$P(X=1) = p$; $P(X=0) = (1-p)$

FAIR coin: $p = 1/2$,

UNFAIR COIN: $p \in [0, 1]$

→ continuous-valued random variables. ←

- Sample space $= [a, b]$, continuous range

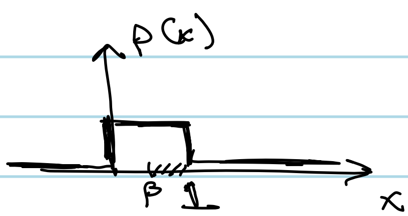


- proba. distribution given by probability density $p(x)$:

$$P(x \in [l, r]) = \int_l^r p(x)\, dx$$

Ex| Uniformly distributed random variable with range $[0, 1]$.

$$p(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & \text{otherwise.} \end{cases}$$



$$p(X > \beta) = 1 - \beta$$

- Generating R.V. in MATLAB

>> x = rand → unif. distributed R.V. w/ range [0,1]

- See what neighbor got!

→ MATLAB generates seq. of pseudo-random vars. w/ range [0,1]. Huge period > 2^{1492}

Need to... "randomize" starting point through seq.

```
>> rand ('state', sum (100 * clock))
```

- Generating a LIST of r.v. : >> xlist = rand (1, n)

— · —

- Generating binary r.v. w/ $p(X=1) = 1/2$
  $p(X=0) = 1/2$

```
>> x = round (rand)
```

$p(x=1) = p$
$p(x=0) = 1-p$

$$>> x = \text{round}\,(\text{rand} + \underbrace{(p - 1/2)}_{\text{Make up for more-than-fair proba.}})$$

- Back to our example. generate_simple_spiketrain.m

[in lecture codes + website]

```
nsec = 1 ;                  number of seconds
T = 1 ;                     total number of seconds
deltat = 0.001 ;            1 msec bins
r = 100 ;                   rate (spikes per sec, Hz)
p = r * deltat ;

numbins = round(T/deltat)
spiketrain = round(rand(1,numbins) + (p - 1/2))
```

"SAMPLE"

BASIC STATISTICS OF R.V. X:

- Say... Have M samples (realizations), $X = s_j$ "on trial j"

- Mean $(X) = \mathbb{E}(X) = \langle X \rangle = \overline{X} =$

$$\approx \frac{1}{M} \sum_j s_j$$

- $Var(X) =$ squared fluctuations in samples around $\langle X \rangle$

$$= \mathbb{E}\left(\left(X - \langle X \rangle \right)^2 \right)$$

$$\approx \frac{1}{M} \sum_{j=1}^{M} \left(s_j - \left(\frac{1}{M} \sum_{j=1}^{M} s_j \right) \right)^2$$

- Fact: these sample stats $\rightarrow$ true stats (eg, $\mathbb{E}X = \sum_j s_j \, p(s_j)$) in lim $M \rightarrow \infty$. [laws of lg. #'s]

- Implementation in MATLAB:

given sample-list $= (s_1, \ldots, s_M)$

mean (sample-list)
var (")

→ Mean, var. of spike count across trials, in last 1/2 of generate-simple-spiketrain.m ←